MA261 Quiz 10

July 29, 2016

Problem 1.

Find the tangent plane of a parametric surface

$$\mathbf{r}(u, v) = (u^2 - 4v)\,\mathbf{i} + (v^2 - 2v)\,\mathbf{j} + (u - 2v)\,\mathbf{k}$$

at the given point $(0, -1, 0)$. (That is, when $u = 2$ and $v = 1$.)
(**Hint**: The normal vector of the tangent plane is $\mathbf{r}_u \times \mathbf{r}_v$.)

Solution.

$$\begin{aligned}
\mathbf{r}_u &= \langle 2u, 0, 1\rangle = \langle 4, 0, 1\rangle \\
\mathbf{r}_v &= \langle -4, 2v - 2, -2\rangle = \langle -4, 0, -2\rangle \\
\mathbf{r}_u \times \mathbf{r}_v &= \langle 0, 4, 0\rangle
\end{aligned}$$

The tangent plane is $0 = 4(y + 1)$, or simply $y = -1$.

Problem 2.

Evaluate the surface integral

$$\int\int_S y - z\, dS$$

where S is a surface with parametric equations

$$\begin{cases} x &= u^2/2 \\ y &= u + v \quad (0 \le u \le 2,\ 0 \le v \le 1) \\ z &= v \end{cases}$$

(**Hint**: $dS = |\mathbf{r}_u \times \mathbf{r}_v| dA$)

Solution.

$$\mathbf{r}_u = \langle u, 1, 0 \rangle$$
$$\mathbf{r}_v = \langle 0, 1, 1 \rangle$$
$$\mathbf{r}_u \times \mathbf{r}_v = \langle 1, -u, u \rangle$$
$$|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{1 + 2u^2}$$
$$dS = |\mathbf{r}_u \times \mathbf{r}_v| dA = \sqrt{1 + 2u^2} du\, dv$$
$$\int \int_S y - z\, dS = \int_0^1 \int_0^2 u\, \sqrt{1 + 2u^2} du\, dv$$
$$= \frac{27 - 1}{6} = \frac{13}{3}$$